UNIVERSITE LIBANAISE
Institut Universitaire de Technologie

الجامعة اللبنانية
المعهد الجامعي للتكنولوجيا

Entrance Exam: September 10, 2013

Mathematics - GC - GIM - GRIT Duration: 2 H

N.B.: All questions are obligatory

Exercise 1. (4 Pts)

A house has a water tank with a capacity of 2 m^3. Initially it is empty. The tank is supplied with a flow rate of 0.1 liter/second starting at midnight. The water consumption in the house is constant of 0.15 liter/second. This consumption begins at 5am and continues until 6pm.
Will the house be out of water? If yes, at what time? Explain your answer.

Exercise 2. (6 Pts)

To produce 1 m^3 of concrete we need 0.8 m^3 of gravel, 0.4 m^3 of sand and 300 kg of cement. The prices of these materials are shown in the following table:

	m^3 of gravel	m^3 of sand	1000 kg of cement
2012	15U SD	30 USD	120 USD
2013	20 USD	40 USD	140 USD

a) *What is the rate of increase in the cost per m^3 of concrete in 2013 compared to 2012?*
b) *What is the cost of a concrete column with the following dimensions 80 x 20 x 330 (en cm) using 2013 prices?*
c) *What will be the cost of this column in 2016 using the same increasing rate calculated before?*

Exercise 3. (10 Pts)

Consider the equation (E) : $z^3 + (2-i)z^2 + (5-2i)z - 5i = 0$, where the unknown z belongs to the set of complex numbers.

1) Determine the real number α so that $z = \alpha i$ is a solution of (E).
2) Determine the real numbers a and b so that:
$$z^3 + (2-i)z^2 + (5-2i)z - 5i = (z-i)(z^2 + az + b)$$
3) Solve (E).
4) Let A, B and C be the points of affixes i, -1 +2i and -1 -2i respectively.
   a- Find the affix of point D on the x-axis so that A,B and D would be collinear.
   b- Calculate $\dfrac{z_D - z_B}{z_D - z_C}$. Deduce the nature of triangle BCD.
   c- Find the affix of point E such that BCE is an equilateral triangle.

Exercise 4. (10 Pts)

For **a period of 40 days**, Maha had registered the number of email messages that she has received daily. The results are shown in the following table:

Number of email messages per day	0	1	2	3	4	5	6	7	8
Number of days	1	4	3	8	7	7	5	2	3

1) Consider the events:
   A: " Maha receives exactly 3 email messages per day"
   F: " Maha receives more than 5 email messages per day".

Verify that $P(A) = \frac{1}{5}$ and $P(F) = \frac{1}{4}$.

2) Consider the event
S:" Maha receives an email message from her friend Sanaa".
If Maha receives more than 5 email messages per day, then the probability of receiving an email message from her friend Sanaa is 0.3. If Maha receives 5 email messages or less per day, then the probability of receiving an email message from her friend Sanaa is 0.2.
Calculate the probabilities $P(S \cap F)$, $P(S \cap \overline{F})$. Deduce P(S).

3) In the following, assume that the number of email messages that Maha received daily has **doubled**.
a- Find the average number $\overline{x}$ and the median of email messages received daily by Maha.
b- Calculate the standard deviation σ. What percentage of the data set is included in the interval $\left[\overline{x}-\sigma, \overline{x}+\sigma\right]$..

Exercice 5. (15 Pts)

Let f be the function defined, on $I =]0 ; +\infty[$, by $f(x) = x + 1 + \ln\left(\frac{x}{x+1}\right)$. (C) is the representative curve of f in an orthonormal system $(O, \vec{i} ; \vec{j})$; (unit 2cm).

1) a- Prove that the line of equation x = 0 is an asymptote of (C)
b- Calculate $\lim_{x\to+\infty} f(x)$. Prove that the line (d) with equation y = x+1 is an asymptote of (C).
c- Study the relative position of (C) and (d).
2) Show that f is strictly increasing on I, and set up its table of variations.
3) Prove that the equation f (x) =0 has a unique root α and verify that 0.3 < α < 0.4.
4) Draw, (d) and (C).
5) Designate by g the inverse function of f and by (G) its representative curve.
a- Deduce the asymptotes of (G) and set up the table of variations of g.
b- The curve (C) and (G) have a point A in common. Determine the coordinates of A.

6) a- Verify that the function F defined, on I, by:
$$F(x) = \frac{x^2}{2} + x + x\ln x - (x+1)\ln(x+1)$$ is an antiderivative of f.
b- Calculate, in cm^2, the area of the region bounded by the curve (C), the x-axis and the two lines of equations x = 1 and x = 2.

Exercice 6. (10 Pts)

In the space referred to an orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points A(0 ; 1 ; – 2) , B(2 ; 1 ; 0) , C(3 ; 0 ; – 3) and H(2 ; 2 ; – 2).
1) Show that x – 2y – z = 0 is an equation of the plane (P) determined by the points H , A and B and that the point C does not belong to this plane.
2) a- Show that triangle HAB is isosceles of vertex H.
b- Show that (CH) is perpendicular to (P).
c- Determine a system of parametric equations of a bisector (δ) of angle $A\hat{C}B$.
3) Let T be the orthogonal projection of H on plane (ABC); prove that T belongs to (δ)

Exercice 7. (5 Pts)

a) Solve the differential equation (E): y'' + 2y' + y = 0.
b) Determine the particular solution of (E) whose representative curve, in an orthonormal system, is tangent at the point of abscissa 0 to the line of equation: y = – x + 2.