Lebanese University
University Institute of Technology

الجامعة اللبنانية
المعهد الجامعي للتكنولوجيا

Entrance Exam: September 8, 2016

Mathematics: CCNE - IEM - CE Time: 2 Hours

N.B.: All questions are obligatory

Exercise 1 (5 Pts)

Let the plane be reported to an orthonormal system $(O;\vec{u},\vec{v})$.

a) Solve in the set of complex numbers the equation: $z^2 - 4z + 6 = 0$.

b) Let A and M be the points of affixes $z_A = 3$ and $z_M = 2 + i\sqrt{2}$ respectively.
Determine the algebraic form of the complex number $\frac{z_M - 3}{z_M}$.

c) Deduce that the triangle OAM is right.

Exercise 2 (5 Pts)

Consider the numerical sequence (u_n) defined by:

$$u_1 = 12 \quad \text{and} \quad u_{n+1} = \frac{1}{3}u_n + 5 \text{ for all natural integers } n \geq 1.$$

Let the sequence (v_n) be defined by: $v_n = u_n - \frac{15}{2}$, for all natural integers $n \geq 1$.

a) Prove that the sequence (v_n) is a geometric sequence with common ratio $\frac{1}{3}$.

b) Express then v_n in terms of *n*.

c) Determine the limit of the sequence (v_n) and then deduce the limit of the sequence (u_n).

Exercise 3 (5 Pts)

A and **B** are two vaccines. 40% of a given population is vaccinated with **A** and 80% with **B**.
We choose randomly a person from the population. What is the probability that the person:

a) is vaccinated with **A** and **B**?

b) is vaccinated with **A** or **B**?

c) is vaccinated with **A** given **B**?

Exercise 4 (5 Pts)

Calculate the following integrals:

a) $\int_0^1 (x^2 + x - 3)e^{-x}dx$ b) $\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx$ c) $\int_2^3 \frac{x^2 + x - 6}{x - 1}dx$

Exercise 5 (5 Pts)

Consider the following differential equation: $xy' + (1 - x)y = 0$.

a) Let $z = x\,y$. Find the differential equation satisfied by *z*.

b) Deduce the solution *y*.

Exercise 6 (5 Pts)

Discuss graphically the number and the sign of the roots of the equation $4x^3 - 3x - m = 0$, where m is a real parameter.

Exercise 7 (5 Pts)

Let α be an angle such that $0 < \alpha < \frac{\pi}{2}$ with $sin\ \alpha = k$.

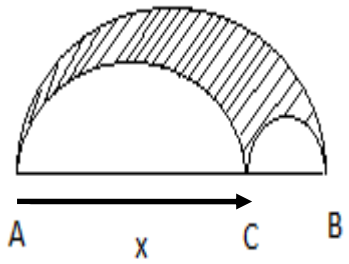
a) Calculate $cos\ \alpha$ in terms of k.

b) Determine, in terms of k, the expression of $E = \sin\left(\frac{13\pi}{2} + \alpha\right) \cos(135\pi - \alpha)$.

c) Deduce the numerical value of *E* for $k = \frac{1}{3}$.

Exercise 8 (5 Pts)

Let [AB] be a segment of length 10 cm and C be a point of [AB] such that AC = *x*.
Denote by *f(x)* the area of the shaded surface of the below figure containing three semicircles.



a) What is the domain of definition of *f*.

b) Show that $f(x) = \frac{\pi}{2}\left(-\frac{x^2}{2} + 5x\right)$.

c) Show that *f(x)* is maximum for a value x_0 to be determined.

Exercise 9 (5 Pts)

A shop decides to sell its stock in the form of lots:

1st lot	: 5 shirts,	10 ties,	1 pair of trousers;
2nd lot	: 8 shirts,	8 ties,	2 pairs of trousers;
3rd lot	: 7 shirts,	7 ties,	5 pairs of trousers;

How many of each lot should he propose to sell 300 shirts, 400 ties and 96 pairs of trousers?
Note: you should solve the system of equations without using the calculator.

Exercise 10 (5 Pts)

Given the points A(0 ; 0 ; 1) and B(2 ; –1 ; 3).

a) Write the equation of the sphere (S) of diameter [AB].

b) Find the set of points M(*x* ; *y* ; *z*) of the space such that $\overrightarrow{AM}.\overrightarrow{AB} = 0$.

Good Luck