Lebanese University
University Institute of Technology

الجامعة اللبنانية
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Entrance Exam (Engineering)
MathematicsExam

July 25, 2017 **Time: 2hours**

N.B.: Questions 1, 2, 3, and 4 are obligatory

Exercise 1 (10 Pts)

Consider the two functions f and g defined over]0 ; +∞[by:

$$f(x) = 2x + \frac{1 - \ln x}{x} \text{ and } \quad g(x) = 2x^2 - 2 + \ln x$$

Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{\imath}, \vec{\jmath})$.

A- 1) Show that g is strictly increasing over]0; +∞[.

2) Calculate $g(1)$ and deduce the sign of g(x) according to the values of x.

B- 1) a- Determine $\lim\limits_{x \to 0} f(x)$and deduce an asymptote to (C).

b- Show that the line (d) with equation$y = 2x$is an asymptote to (C). Study, according to the values of x, the relative positions of (C) and (d).

2) Show that $f'(x) = \frac{g(x)}{x^2}$.

3) Set up the table of variations of f.

4) Draw (d) and (C) in the system $(O; \vec{\imath}, \vec{\jmath})$.

5) a- Show that f has over [1;+∞[an inverse function h whose domain of definition is to be determined.

b- Draw (Γ), the representative curve of h in the same system as that of (C).

c- Determine the abscissa of the point of (Γ) where the tangent is parallel to the line withequation $y = \frac{x}{2}$.

Exercise 2 (10 Pts)

The space is referred to a direct orthonormal system$(O; \vec{\imath}, \vec{\jmath}, \vec{k})$.

Consider the plane (P) with equation x – 2y + 2z – 6 = 0

and the line (d) with parametric equations $\begin{cases} x = m + 1 \\ y = 2m + 1 \\ z = 2m + 2 \end{cases} \quad (m \in IR)$

Let (Q) be the plane containing (d) and perpendicular to (P) andA (1; 1; 2) a point on (d).

1) Show that 2x–z = 0 is an equation of the plane (Q).

2) Prove that the line (Δ)with parametric equations $\begin{cases} x = 2t \\ y = 5t - 3 \\ z = 4t \end{cases} \quad (t \in IR)$

is the line of intersection of (P) and (Q).

3) a-Determinethe coordinates ofB, the meeting point of (d) and (Δ).

b-Determine the coordinates of point F, the orthogonal projection of A on(Δ).

c-Calculate the cosine of the angle formed by (d) and (P).

Exercise 3 (10 Pts)

An urn contains even balls: four red balls and three green balls.

A player selects randomly and simultaneously three balls from this urn.

1) a- Calculate the probability that the player selects exactly two red balls.

   b- Show that the probability that the player selects at least two red balls is equal to $\frac{22}{35}$.

2) After selecting three balls, the player scores:
   - 9 points if he gets three red balls;
   - 6 points if he gets exactly two red balls;
   - 4 points if he gets exactly one red ball;
   - Zero if he gets three green balls.

   Denote by X be the random variable that is equal to the score of the player.

   a- Determine the probability distribution of X.

   b- Knowing that the player scored more than 2 points calculate the probability that his score is multiple of 3.

Exercise 4 (10 Pts)

In the complex plane referred to a direct orthonormal $(O;\vec{u},\vec{v})$, consider the points Mand M' with respective affixes z and z' such that: $z' = (1 + i\sqrt{3})z - 2$.

1) In this part, suppose that z = 1+i.

   a- Show that the point M' belongs to the line with equation y = –x.

   b- Show that triangle OMM' is right at O.

2) Let I be the point with affix –2.

   a- Verify that $|z' + 2| = 2\,|z|$.Deduce that $\left\|\overrightarrow{IM'}\right\| = 2\left\|\overrightarrow{OM}\right\|$.

   b- Prove that as M describes the circle with center O and radius 2, M' describes a fixedcircle whose center and radius are to be determined.

3) Suppose that z = x+iy and z' = x'+iy' where x, y, x' and y' are real numbers.

   a- Express x' and y' in terms of x and y.

   b- Show that if M describes the line with equation $y = -x\sqrt{3}$, then M' describes a straight lineto be determined.

N.B.: Choose 2 of the following 3 questions

Exercise 5 (5 Pts)

Calculate the following integrals:

a) $\int cos^2(x)\,dx$ b) $\int \frac{1}{x^2+4x+8}\,dx$

Exercise 6 (5 Pts)

Let the differential equation (E):$y'x - y = x^3$over]0 ; +∞[.

1) Solve the differential equation: $y'x - y = 0$.
2) Verify that the function $f(x) = \frac{x^3}{2}$is a particular solution of (E).
3) Deduce the general solution of (E).

Exercise 7 (5 Pts)

Find the equation of the parabola, $y = ax^2 + bx + c$, that passes through the following three points: (-2, 40), (1, 7), (3, 15).Note that the equations system must be resolved without calculator.

Good Work