**UNIVERSITE LIBANAISE**
**Institut Universitaire de Technologie**

**الجامعة اللبنانية**
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# Entrance exam: September 10, 2013

**Mathematics - IAG Duration: 2H**

***N.B.: All questions are obligatory***

## Exercise 1. (5 Pts)

The distance between Tripoli and Beirut is 90km. Two cars A and B get off at the same time one from Beirut (A) and the other from Tripoli (B). The car A runs at 100 km/h and B at 80 km/h. After how much time these two cars will meet and how far from Beirut?

## Exercise 2. (7 Pts)

Suha owns a car which is worth 3000 USD. This car consumes 20 liters of gasoline per 100 km in town and 14 liters/100km on highways. Gasoline costs 1500 LL/liter. She travels annually 5000 km in town and 10000 km on highways.
She plans to purchase a new car for 12000 USD all expenses included. This car consumes 12 liters per 100 km of gasoline on town and 8 liters/100 km on highways. After how long this sale will be fully profitable? 1USD = 1500 LL. The cost of maintenance is not taken into consideration

## Exercise 3. (11 Pts)

The following table shows the content and the value of three shopping baskets at the exit of fruit market:

| | Basket 1 | Basket 2 | Basket 3 |
|---|---|---|---|
| Grappe | 4 kg | 1 kg | 3 kg |
| Apple | 2 kg | 4 kg | 1 kg |
| Banana | 3 kg | 2 kg | 1 kg |
| Valeur totale | 18500 LL | 13000 LL | 10000 LL |

a) Construct the system of equations corresponding to the above table.
b) Solve the equations system thus obtained and give the price of each kilo of fruit.

## Exercise 4. (12 Pts)

An engineer started working in January 2010, with an annual salary of 36 million LL. According to his contract, he received in January of every new year an annual increase of 4% on his previous annual salary, to which a constant yearly bonus of 500 000 LL is then added.
Designate by $U_n$ the annual salary, in millions of LL, of this engineer in January 2010+n, $S_o$
$U_0 = 36$.

1. Compute $U_1$. Give the expression of $U_{n+1}$ in terms of $U_n$
2. Let $U_n = V_n + \alpha$ , for all n in IN, where $\alpha$ is a real number, and ($V_n$) is a geometric sequence with common ratio 1.04.
   a) Prove that $\alpha = -12.5$.
   b) Determine the value of $V_0$ and calculate $V_n$ then $U_n$ in terms of n.
3. If this engineer started to save 30% of each of his salaries since January 2010, what is the total amount of money saved by him the first January 2020?

## Exercise 5. (13 Pts)

The following table represents the ages' distribution of 26 men and 24 women.

| Age in years | $[20;25[$ | $[25;30[$ | $[30;35]$ |
|---|---|---|---|
| Number of men | 8 | 8 | 10 |
| Number of women | 5 | 9 | 10 |

A committee of 3 people will be formed out of these 50 people. Consider the following events:

**A**: The committee is mixed (formed of men and women)
**B**: the age of each member of the committee is less than 30.

1) Prove that the probability p(A) = 0.764 and calculate p(B).
2) a- Calculate $p(B\cap\overline{A})$, and deduce $p(B\cap A)$.
   b- Determine $p(B/A)$.
3) C is the event defined by:
   The committee is formed by three women where the age of each member is greater than or equal to 30.
   Calculate p(C).
4) X is the random variable that is equal to the number of youngest women (group $[20,25[$) in the committee. Determine the probability distribution of X.

## Exercise 6. (15 Pts)

Let f be the function defined on $]0;+\infty[$ by $f(x) = \ln x - 1 - \frac{1}{x}$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x\to 0} f(x)$ and interpret the answer.
   b- Calculate $\lim\limits_{x\to +\infty} f(x)$.
2) Calculate f '(x) and set up the table of variations of f.
3) Prove that the equation f(x) = 0 has a unique solution α and verify that $3.59 < \alpha < 3.6$.
4) Prove that (C) does not have points of inflection.
5) Determine an equation of the tangent (T) to the curve (C) at the point of abscissa 1.
6) Draw (T) and (C).
7) Use integration by parts to prove that the area $A$ of the region bounded by the curve (C), the tangent (T) and the axis of abscissas is: $A = (\alpha + \frac{1}{\alpha} - 4)$ units of area.

## Exercise .7 (7 Pts)

Calculate the following integrals :

a) $\int x^2\sqrt{1-x^3}\,dx$ b) $\int_{-1}^{1}\frac{2x-1}{x^2-x-6}\,dx$ c) $\int_0^2 (2x+1)e^x dx$