Lebanese University
University Institute of Technology

الجامعة اللبنانية
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Entrance Exam: September 11, 2015

Mathematics - ABC **Time: 2 hours**

Exercise I (12 points)

A bag S contains eight bills: four of 10 000LL, three of 20 000LL and one of 50 000LL.
Another bag T contains also eight bills: three of 10 000LL and five of 20 000LL.

1) Two bills are drawn, simultaneously and randomly, from bag S.
   Calculate the probability of each of the following events:
   A : « the two drawn bills are of the same category »
   B : « the sum of values of the two drawn bills is 30 000LL ».

2) One of the two bags (S and T) is randomly selected, then two bills are simultaneously and randomly drawn from this bag.
   Consider the following events:
   E : « the chosen bag is S »
   F : « the sum of the values of the two drawn bills is 30 000LL »
   Calculate the probabilities $P(F \cap E)$ and $P(F \cap \overline{E})$. Deduce $P(F)$.

3) We draw, randomly, one bill from the bag S and one bill from the bag T.
   Let X be the random variable equal to the sum of the values of the two drawn bills.
   a- Verify that $P(X = 60\,000) = \dfrac{3}{64}$.
   b- Determine the probability distribution of X and calculate its mean (expected value).

Exercise II (12 points)

The table below shows the blood pressure y_i, according to the weight x_i, of a group of women .

Weight in kg x_i	55	58	60	64	65	70
Blood pressure y_i	13.2	13.5	13.8	14.6	15.2	15.8

1) Calculate the averages $\overline{x}$ and $\overline{y}$ of the two statistical variables x_i and y_i respectively.
2) Represent graphically the scatter plot as well as the center of gravity $G(\overline{x}\ ;\ \overline{y})$ of the points $(x_i\ ;\ y_i)$ in a rectangular system.
3) Write the equation of the regression line $D_{y/x}$ of y in terms of *x* and draw this line in the preceding system.
4) Suppose that the above pattern remains valid for weights of women between 45 and 75 kg. Estimate the blood pressure of a woman weighing 72 kg.
5) Doctors assume that a normal blood pressure for a woman should belong to the interval [12 ;13]. Estimate a corresponding interval to which the weights of women having normal blood pressure should belong.

Exercise III (12 points)

A factory produced 3500 tons of cement in the year 2000. The production then declined steadily by 15% per year until the end of 2010. We note U_n the production in tons during the year (2000 + n) and $U_0 = 3500$.

1) Calculate U_1 and U_2.
2) a- Show that the sequence (U_n) is geometric and determine its common ratio.
   b- Express, for $n \leq 10$, U_n in terms of n and calculate the production during the year 2010.
3) After 2010, the production of this factory increased regularly by 15% per year.
   a) Calculate U_{11}.
   b) Determine, in terms of n, the expression of U_n for $n \geq 11$.
   c) From which year, will the annual production of the factory be greater than or equal to that of the year 2000?

Exercise IV (24 points)

Let f be a function defined, on $[0 ; +\infty[$, by : $f(x) = x + 1 + e^{-x+1}$ and designate by (C) its representative curve in an orthonormal system $(O ; \vec{i} , \vec{j})$.

A-1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$.

b- Determine the asymptote to (C).

2) Calculate f '(x) and set up the table of variations of f.
3) Draw (d) and (C).
4) Show that the equation f(x) = 4 has a unique root α and verify that: $2.84 < \alpha < 2.86$.
5) Calculate the area of the region bound by the curve (C), its asymptote (d) and the two lines of equations $x = 0$ and $x = 1$.

B- For the following, let $\alpha = 2.85$.

A factory produces x thousand of toys; $(1 \leq x \leq 5)$.

The cost of production, in millions of LL, is given by : $C(x)=x+1+e^{-x+1}$.

1) Calculate the cost of production of 2 thousand toys. In this case, what is the cost of production of one toy?
2) Find the number of toys that should be produced so that the cost of production will be 4 million LL.

Exercise V (4 points)

A parking lot has spaces reserved for three different types of vehicles: Motorcycles, Cars, and Vans. There are five more spaces reserved for Vans than for Motorcycles. There are three times as many Car spaces as the sum of Van and Motorcycle spaces combined. If the parking lot has 180 total reserved spaces, how many spaces are there for each type?

Exercise VI (4 points)

1) Solve in $\mathbb{C}$ the equation: $z^2 + z + 1 = 0$.
2) Give the obtained values in exponential form.

Exercise VII (6 points) calculate the following integrals:

1) $\int (x+1)e^x dx$ 2) $\int \frac{dx}{x^2-7x+6}$ 3) $\int \frac{\ln(x+2)}{x+2} dx$