Lebanese University
University Institute of Technology

الجامعة اللبنانية
المعهد الجامعي للتكنولوجيا

Entrance Exam: 9 September 2016

Mathematics: Business Computer **Time: 2 hours**

N.B.: Choose 3 exercises from the exercises 1,2,3,4 (exercise 5 is obligatory)

Exercise 1 (16 Pts)

A touristic site proposes two possibilities for visiting: a walking tour, or a bus tour. A refreshment bar selling only one type of drink is installed on the site. Assume that a tourist can buy at most one drink during his visit.

A tourist visits the site. It is established that:

- The probability that he visits on foot is 0.3.
- The probability that he buys a drink knowing that he visits by bus is 0.8.
- The probability that he buys a drink knowing that he visits on foot is 0.6.

Let: C be the event "the tourist visits using the bus"; B the event "the tourist buys a drink".

1. Represent the situation by a tree diagram.
2. What is the probability that the tourist chooses a walking tour and buys a drink? Deduce that p(B)=0.74.
3. The tourist buys a drink. What is the probability that he visited on foot?

The entry ticket costs 4000 L.P. The walking tour is done without additional expenses. The visit using the bus is done with additional expenses of 3000L.P. The price of a drink is 2000L.P per unit. Let X be the random variable representing the expenditure associated with the visit of the tourist.

4. What are the possible values (realizations) of X? Give the probability law of X.
5. What is the mean amount of money collected by the site given that 1000 tourists were received during a certain day?

Exercise 2 (16 Pts)

90 employees work in two factories A and B producing similar products (40 employees in A and 50 in B). The following table shows the distribution of monthly salaries (in 100 000 L.P.) :

Salaries (in 100 000 L.P.)	[4; 8[	[8 ;12 [	[12; 16[	[16 ; 20[	[20; 24]
Number of employees of factory A	10	9	12	7	2
Number of employees of factory B	9	16	14	10	1

1. Calculate the means and the standard deviations of the two sets of data A and B.
2. Which one of the two sets is less dispersed around the mean?
3. What is the mean salary of the 90 employees?
4. The community chooses, at random, an employee. Consider the following events:
   - E : « The employee chosen earns at least 1 200 000 L.P. monthly ».
   - F : « The employee chosen works in factory A ».
   - G : « The employee chosen works in factory B ».

   a. Calculate the following probabilities: $p(E/F)$, $p(F)$ and $p(E \cap F)$.

   b. Calculate $p(E \cap G)$ and deduce $p(E)$.

Exercise 3 (16 Pts)

A person deposits on the first of January of the year 2000, an amount of 5000 dollars in a bank at an annual interest rate of 6% (so the annual interest on an amount x will be 0.06*x). The person has the ability of saving 3000 dollars per year to be added to his account on the first of January of each year.

Denote by S_n the amount the person will possess on the first of January the year 2000+n.

1. Calculate S_0, $S_1$and S_2.
2. Show that $S_{n+1} = 1.06S_n + 3000$.
3. Suppose $T_n = S_n + 50000$.
    a. Show that (T_n) is a geometric sequence whose first term and common ratio are to be determined.
    b. Express T_n then $S_n$in terms of n.
    c. On the first of January of which year will the person possess for the first time a sum that exceeds 50000 dollars?

Exercise 4 (16 Pts)

Calculate each of the following integrals:

1. $\int_{-1}^{0}(3x+2)^4 dx$
2. $\int_0^2 \frac{xdx}{\sqrt{2x^2+1}}$
3. $\int_0^3 |2x-4|\, dx$
4. $\int_0^1 x\sqrt{x+1}dx$ (using integration by parts)
5. $\int_3^4 \frac{1}{x^2-3x+2}dx$ (using $\frac{1}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2}$).

Exercise 5 (32 Pts, obligatory)

Part A: Consider the function f defined over $I = [0, +\infty[$ by $f(x) = 0.4x + e^{(-0.4x+1)}$ and designate by (C) its representative curve in an orthonormal system.

1. Determine the limit of $f(x)$ at $+\infty$. Show that the straight line (d) of equation $y = 0.4x$ is an asymptote to (C) and study the relative positions of (C) with respect to (d).
2. Solve, in the interval I the inequality $1 - e^{(-0.4x+1)} > 0$ and study the variations of $f(x)$ over the interval I. Draw its table of variations and deduce the sign of $f(x)$ over $[0, +\infty[$.
3. Show that the tangent (T) to (C) at the point of abscissa 0, passes through the point B(2.5 ;1). Draw (d), (T), and (C).

Part B : let x be the number of articles, expressed in hundreds, produced by a factory, and $f(x)$ the total cost of production, expressed in thousands of dollars. Assume that all the items are sold

4. Each article is sold for 5 dollars. Express the revenue$R(x)$, in thousands of dollars, in terms of x.
5. Construct on the preceding graph, the curve (Δ), representative of the function R, and verify graphically that (C) and (Δ) intersect at a unique point of abscissa α such that $4.49 < \alpha < 4.5$.
6. Show that the profit $P(x)$, is expressed in thousands of dollars by $P(x) = 0.1x - e^{(-0.4x+1)}$.
7. Study the variations of P over $I = [0, +\infty[$ and draw its representative curve. Deduce the minimum number of articles to be produced in order to realize profit.

Good luck