Lebanese University
University Institute of Technology

الجامعة اللبنانية
المعهد الجامعي للتكنولوجيا

Entrance Exam:26 July 2017

Mathematics: Business Computer **Time: 2 hours**

Exercises 1, 2 and 3 are obligatory. Choose 2 exercises from the exercises 4, 5 and 6.

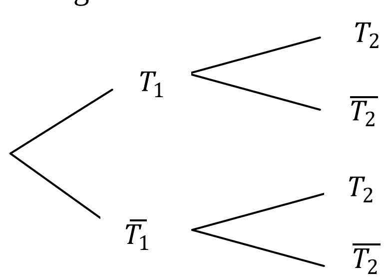
Exercise 1 (16 Pts, obligatory)

In a bicycle tire manufacturing plant, each product must pass two tests (T1) and (T2). A quality study showed that:

- 92% of the tires pass the test (T1).
- 95% of the tires that passed the test (T1), also pass the test (T2).
- 25% of those who failed the test (T1), also fail the test (T2).

A tire is selected at random and the following events are considered:
T_1 = "the tire passes the test (T1)"
T_2 = "the tire passes the test (T2)".

The different situations are modeled by the following tree:



1. Give the probability values on each branch of the tree.
2. Calculate the probability that the tire does not pass both tests: $P(\overline{T_1} \cap \overline{T_2})$.
3. Calculate the probabilities $P(T_2 \cap T_1)$ and $P(T_2 \cap \overline{T_1})$. Deduce that $P(T_2) = 0.934$.
4. A selected tire passes the test (T2). What is the probability that he will also pass the test (T1)?
5. When the tire passes both tests, it is sold at 16000L.L. The tire passing a single test will be sold at 8000L.L. And tires failing both tests will be donated free of charge to a recycling plant. Let X be the random variable representing the price of a product tire.
    a. What are the possible values of X?
    b. Give the probability distribution of X.
    c. Calculate the mathematical expectation (the mean) of X.
    d. What should be the number of tires manufactured so that the factory income will be equal to 2669760L.L.?

Let f and g be the functions defined on $[0;+\infty[$ by $f(t) = 2\ln(t+1)+1$ and $g(t) = \frac{4}{1+e^{-t}}$.

1. Give the limits of f and g at $+\infty$.
2. Draw the table of variation for f and g.
3. Give the equation of the horizontal asymptote for the function g.
4. Draw the curves (C) and (G) of f and g respectively
5. It is assumed that the curves (C) and (G) intersect at a point of abscissa $\alpha \approx 3.1$. Study the sign of $g(t) - f(t)$ graphically according to the values of t.
6. Calculating primitives:
   a. Show that $g(t) = \frac{4e^t}{e^t+1}$for every t in $[0;+\infty[$. Deduce a primitive of g over $[0;+\infty[$.
   b. Let H be the function defined on $[0;+\infty[$ by $H(t) = (t+1)\ln(1+t) - t$. Determine the derivative of H and deduce a primitive of f over $[0;+\infty[$.
7. A restructuring plan in an industry is established over 5 years. It is assumed that f (t) models the number of jobs created (in thousands of jobs), and that g (t) models the number of jobs eliminated (in thousands of jobs) as a function of t representing time in years.
   a. Determine the time needed so that the number of jobs created exceeds the number of jobs eliminated?
   b. It is assumed that over 5 years, the variation in the number of jobs is given by $I = \int_0^5 (f(t) - gtdt$. Calculate I and deduce if there are more jobs created or eliminated over the 5 years?

Exercise 3 (15 Pts)

Calculate each of the following integrals:

1. $\int_{-1}^{2} x(3x^2+1)^3 dx$
2. $\int_1^2 \frac{(3-\ln(x))dx}{x}$
3. $\int_{-2}^{3} |x-2|\, dx$
4. $\int_0^1 (2x+1)e^{-x} dx$ (using integration by parts)
5. $\int_3^4 \frac{x-1}{x^2-2x+2} dx$.

Choose 2 exercises from the exercises 4, 5 and 6.

Exercice 4 (15 Pts)

A honey producer produces 120 liters every Saturday, which he stores in a barrel with a maximum capacity of 300 liters. Assuming that over a long period, the sale of honey is exactly 75 percent of the existing quantity. Let V_n be the volume stored on the nth Saturday after the harvest.

1. Show that $V_3 = 157{,}5$ liters and calculate volume V_4.
2. Express V_{n+1}as a function of V_n.

For every positive integer n, we define t_n by : $t_n = 160 - V_n$.

Exercise 2 (19 Pts, obligatory)

3. Show that $(t_n)$is the geometric sequence of first term $t_1 = 40$ and of common ration ¼.
4. Deduce the expressions of t_n then of V_n as a function of n.

5. Determine the limit of (t_n) then that of (V_n).
6. Is the barrel large enough to store the honey harvested every Saturday?

Exercise 5 (15 Pts)

Parts A and B are independent
Part A
Nabil Decides to open a savings account at a bank giving a compound interest with a monthly interest rate of 0.4% compounded monthly (so the monthly interest calculated for an amount x equals 0.004 * x). Nabil deposits the sum of 10,000 dollars at the beginning of January 2006, and starting from the first of February, he deposits 600 dollars to his account each month. Let us denote by u_n the amount that Nabil will possess after n months. We give $u_0 = 10000$ and $u_1 = 10640$.

1. Calculate u_2 *and* u_3, and find a relation between u_{n+1} and u_n.
2. Let (v_n) be the sequence defined for all n by: $v_n = u_n + 150000$.
   a. Show that (v_n) is a geometric sequence and give the first term and the common ratio.
   b. Calculate v_n and then u_n as a function of n.
   c. What is the time required for Nabil to have 100,000 Dollars in his account?

Part B
Nabil wants to invest a sum of 10,000,000L.L. during 10 years and must choose between two offers:

Offer A: Invest the sum in a bank giving compound interest at an annual interest rate of 8%, compounded semiannually.
Offer B: Invest the sum in another bank giving compound interest at an annual interest rate of 8%, compounded monthly.
Which of the two offers is more advantageous for Nabil?

Exercise 6 (15 Pts)

The complexe plan is reported to a direct orthonormal system $(O;\ \vec{u},\vec{v})$. Let A be the point of affix $Z_A = -i$ and B the point of affix $Z_B = -2i$. Let f be the application that, for every point M of affix Z, M distinct from A, associates the point M' of affix Z' defined by $Z' = \frac{iZ-2}{Z+i}$.

1. Prove that if Z is pure imaginary, $Z \neq -i$, then Z' is pure imaginary.
2. Determine the invariant points by the application f.
3. For $Z \neq -i$, calculate $|Z' - i| \times |Z + i|$. Show that when the point M moves on the circle of center A and radius 2, the point M' will belong to a circle whose center and radius are to be determined
4. Expand $(Z + i)^2$ and factorize $Z^2 + 2iZ - 2$. Then, determine the set of points M, such that M' is the symmetric of M with respect to O.
5. Determine the set of points E such that the modulus of Z' is equal to 1 (we can observe that $Z' = \frac{i(Z-Z_B)}{Z-Z_A}$).

Good luck