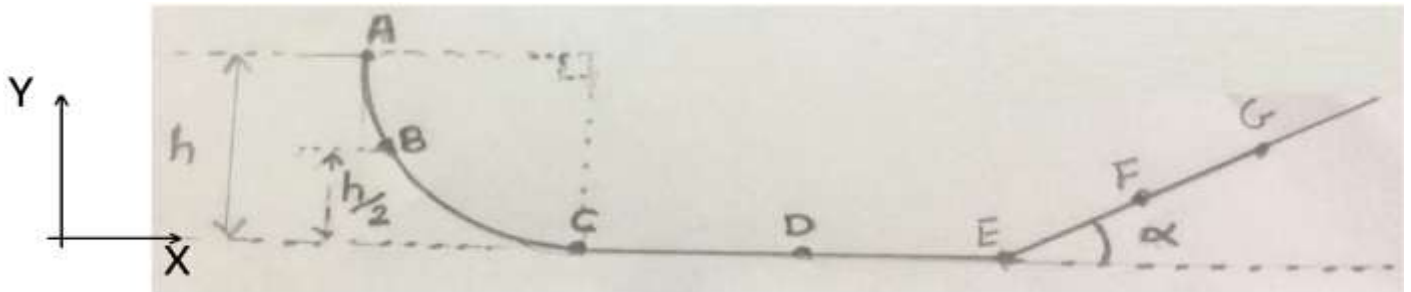


Entrance Exam: September 10, 2013
Physics

Duration: 2 H

Exercise I: (12 points)

A metallic solid S, considered as punctual, is dropped, from a height h without initial velocity, on a frictionless track where the trajectory ABC is circular (1/4 of a circle) of radius $R=h$ (figure below). After it passes point C, the solid S continues its movement with constant friction, first on the horizontal plan (trajectory CDE) then on the inclined plan at an angle α before stopping at point G.



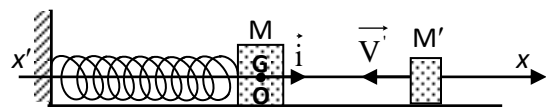
We give: the mass of the solid S is $m=0.1\text{Kg}$; $h=5\text{m}$; the velocity at the point E of S is $V_E=8\text{m/s}$; $\alpha=30^\circ$; gravity $g=10\text{m/s}^2$

The level of the horizontal plan (soil level) is considered as the level of reference of the potential energy.

- 1) Calculate the kinetic energy of the solid S at the moment of its crossing point B, where point B is at $h_1=h/2$. Deduce the values of the components V_{BX} (horizontal) and V_{BY} (vertical) of the velocity of S at point B.
- 2) Calculate the kinetic energy of the solid S at the moment of its crossing point D of the horizontal plan such as $CD=CE/2$.
- 3) Calculate the distance EG passed by S on the inclined plan.
- 4) Calculate the mechanical energy of the system (S, soil) when the solid S passes by the point F of the inclined plan such as $EF=EG/2$.

Exercise II: (12 points)

A spring of un-jointed loops, of stiffness constant $k=10\text{N/m}$ and of horizontal axis, is fixed from one extremity to a fixed obstacle; the other extremity is attached to a puck M of mass $m=100\text{g}$. The center of inertia G of M can slide, without friction, along a horizontal axis $x'x$ of origin O and unit vector \vec{i} . The horizontal plane passing through G is taken as a gravitational potential energy reference.



At the instant $t_0=0$, the puck M, initially at rest at O, is hit with another puck M' of mass $m'=\frac{m}{2}$ moving

initially with a velocity $\vec{V}'=-V'\vec{i}$ ($V'>0$). After collision, the puck M' rebounds on M with a velocity \vec{V}_1 and the puck M moves with a velocity $\vec{V}_0=V_0\vec{i}$, and performs oscillations with a constant amplitude $X_m=10\text{cm}$.

- 1) Give the sign of V_0 .
- 2) Let x and v be respectively the algebraic values of the abscissa and the velocity of G at an instant t after the collision.
 - a) Write, in terms of x , m , k and v , the expression of the mechanical energy of the system (M , spring, Earth) at the instant t .
 - b) Derive the differential equation of second order in x that describes the motion of M .
 - c) The solution of this differential equation is of the form $x = A \sin(\omega_0 t + \varphi)$. Determine the values of the positive constants A , ω_0 and φ .
 - d) Deduce that the magnitude of the velocity \vec{V}_0 of M just after the collision is 1 m/s.
- 3) Knowing that the collision between M' and M is supposed to be perfectly elastic, determine:
 - a) the value V' of the velocity of M' before collision;
 - b) the velocity \vec{V}'_1 of M' just after the collision.

Exercise III: (10 points)

The circuit of figure 1 is formed of a function generator (LFG) delivering across its terminals an alternating sinusoidal voltage of frequency f , a coil of inductance $L = 0.07$ H and of negligible resistance, a resistor of resistance $R = 100$ K Ω and a capacitor of capacitance C . The voltage across the LFG is $u_{AM} = U_m \sin \omega t$. The circuit thus carries an instantaneous current given by: $i = I_m \sin (\omega t + \varphi)$

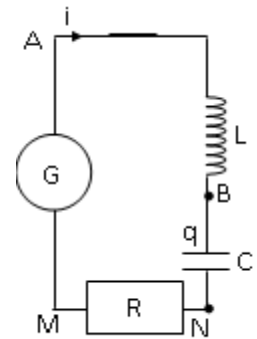


Figure 1

1) We denote by $u_C = u_{BN}$ the instantaneous voltage across the capacitor, by u_{AB} the voltage across the coil and by u_{NM} that across the resistor. Show that:

- a) $i = C \frac{du_C}{dt}$
- b) u_C may be written in the form: $u_C = \frac{-I_m}{C\omega} \cos (\omega t + \varphi)$.
- c) $u_{AB} = L \omega I_m \cos (\omega t + \varphi)$.

2) The relation: $u_{AM} = u_{AB} + u_{BN} + u_{NM}$ is valid for any t . Show, giving ωt a particular value, that:

$$\tan \varphi = \frac{1}{R} - L\omega$$

3) An oscilloscope, conveniently connected, displays the variations, as a function of time, of u_{AM} and u_{NM} on the channels (Y_1) and (Y_2) respectively. These variations are represented in the waveforms of figure 2.

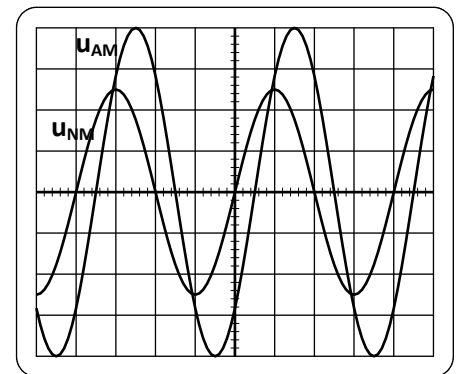
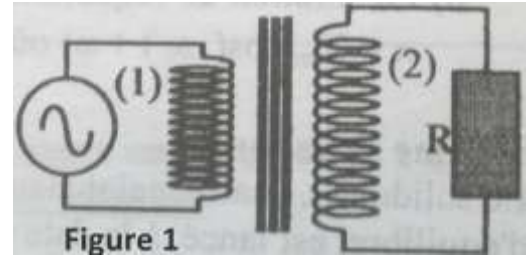


Figure 2

- a) Redraw figure 1 showing the connections of the oscilloscope.
 - b) The waveform of u_{NM} represents the « image » of the current i . Why?
 - c) Find the value of the frequency f , knowing that the horizontal sensitivity is 5ms/division.
 - d) Determine the phase difference φ between i and u_{AM} .
- 4) Deduce the value of the capacitance C .
- 5) The frequency f is made to vary, keeping the same effective value of u_{AM} . It is noticed that, for a value f_1 of f , u_{AM} is in phase with i .
- a) Give the name of the phenomenon that appears in the circuit.
 - b) Deduce, from what preceded, the relation among L , C and f_1 .

Exercise VI: (6 points)

1) The Figure 1 represents the diagram of a loaded transformer. The generator delivers an alternating sinusoidal voltage of frequency f . The coil (1) carries an alternating sinusoidal current i_1 of frequency f . The coil (2) thus carries an alternating sinusoidal current i_2 having the same frequency f . Explain the existence of the current in coil (2).



2) The object of this part is to show evidence of the role of a transformer in the transmission of electric energy. An electric generator G delivers a power $P = 20 \text{ kW}$ under an alternating sinusoidal voltage of effective value U . A transmission line of total resistance $r = 1 \Omega$ feeds an electric installation (B). Let I be the effective current that passes in the line. The power factor of the system formed of the line and the installation is $\cos\varphi = 0.95$.

- a) Give the expression of the power P in terms of U , I and $\cos\varphi$.
- b) i) Give the expression of the power P' lost in the

line due to Joule's effect in terms of P , r , $\cos\varphi$ and U .

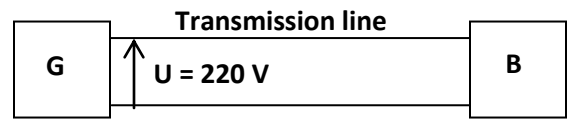


Figure 2

- ii) Calculate P' in the case when $U = 220 \text{ V}$ (Figure 2)

- iii) A transformer, connected across the generator, raises the effective value of the voltage across the transmission line. The transmission of the same power P through the line thus takes place under the new effective voltage $U = 10^4 \text{ V}$ (Figure 3).

Calculate the new value of P' .

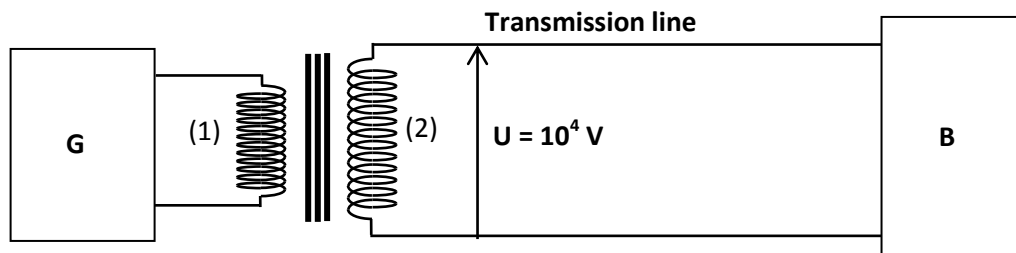


Figure 3

- c) Draw a conclusion about the importance of the usage of the transformer in the transmission of electric energy over large distances.